Math 3236 Statistical Theory

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Pareto DisTr.

$$f(x \mid x_m, A) = \frac{A\, x_m^{A}}{x^{A+1}} \qquad x > x_m$$

$A > 0$

$\mathbb{E}(X)$ is well defined if $A > 1$

$\mathbb{E}(X^2)$ " " " " $A > 2$

$$\mathbb{E}(X) = \int_{x_m}^{\infty} \frac{A\, x_m^{A}}{x^{A}}\, dx = \frac{A}{A+1} \frac{x_m^{A}}{x_m^{A+1}} = \frac{A}{(A+1)\, x_m}$$

a) Wealthiest quintile own $0.2^{\frac{A-1}{A}}$ fraction of The Total wealth.

$$\int_{q(0.8)}^{\infty} f(x)\,dx = 0.8$$

$$\frac{x_m}{q(0.8)} = (0.2)^{1/A}$$

N individuals

$$W = N \int_{x_m}^{\infty} x f(x)\,dx = \frac{NA}{A-1} x_m$$

$$W(0.8) = N \int_{q(0.8)}^{\infty} x f(x)\,dx = N \frac{A}{A-1} \left(\frac{x_m}{q(0.8)}\right)^{A-1}$$

$$\frac{W(0.8)}{W} = (0.2)^{\frac{A-1}{A}}$$

$A = 1.16$ 80 - 20 rule

X_i coming from a pareto distribution, assume x_m is Known.

$\Gamma(\alpha, \beta)$ is a conjugated family to Pareto

$$g_0(A) \simeq A^{\alpha-1} e^{-\beta A}$$

$$g_1(A|x) \simeq A^{\alpha} e^{-\beta A} \left(\frac{x_m}{x}\right)^{A} \frac{1}{x} \simeq$$

$$\simeq A^{\alpha} e^{-\beta A - \log\left(\frac{x}{x_m}\right) A}$$

$$\alpha \to \alpha + 1$$

$$\beta \to \beta + \log\left(\frac{x}{x_m}\right)$$

Size N

$$\alpha \to \alpha + N$$

$$\beta \to \beta + \sum_{i=1}^{N} \log\left(\frac{x_i}{x_m}\right)$$

$$\hat{A}_B(\underline{X}) = \frac{\alpha + N}{\beta + \sum_{i=1}^{N} \log \frac{x_i}{x_m}}$$

ML.

$$L(A) = \prod_{i=1}^{N} f(x_i \mid A) =$$

$$= A^N \prod_{i=1}^{N} \left(\frac{x_m}{x_i}\right)^{A+1} x_m^{-N}$$

$$\log L(A) = N \log A + \sum_{i=1}^{N} \log\left(\frac{x_m}{x_i}\right)(A+1) - N x_m$$

$$\frac{N}{A} + \sum_{i=1}^{N} \log\left(\frac{x_m}{x_i}\right) = 0$$

$$\hat{A}_{ML}(\underline{X}) = \frac{N}{\sum_{i=1}^{N} \log \frac{x_i}{x_m}}$$

$\alpha = 0 \quad \beta = 0 \qquad \hat{A}_B(\underline{X}) = \hat{A}_{ML}(\underline{X})$

improper prior.

$$\mathbb{E}(X) = \frac{A}{A-1} x_m$$

$$\frac{\hat{A}_M}{\hat{A}_M - 1} x_m = \overline{X}$$

$$\hat{A}_M(\underline{X}) = \frac{\overline{X}_n}{\overline{X}_n - x_m}$$

M. M. is consistent.

$\frac{x}{x - x_m}$ is continuous if $x > x_m$

$$\hat{A}_M \xrightarrow{P} A$$

$$\hat{A}_M - A \xrightarrow{d} N\left(0, \frac{A(A-1)^2}{N(A-2)}\right)$$

Variance of $\hat{A}_{ML}$

$$-\mathbb{E}\left(\partial^2_A \log (f(X \mid A))\right)$$

$$\log f(X \mid A) = \log A + A \log x_m - (A+1) \log x$$

$$\partial^2_A f(X \mid A) = -\frac{1}{A^2}$$

$$I(A) = \frac{1}{A^2}$$

$$\hat{A}_{MLE} - A \xrightarrow{d} \mathcal{N}\left(0, \frac{A^2}{N}\right)$$

$$\frac{1}{N} \sum_i \log\left(\frac{X_i}{x_m}\right) \rightarrow \mathcal{N}(??, ??)$$

$$Var(\hat{A}_{ML}) = \frac{A^2}{N}$$

$$Var(\hat{A}_{M}) = \frac{A(A-1)^2}{N(A-2)}$$

$$\frac{A(A-1)^2}{A-2} > A^2 \qquad \text{if} \quad A > 2$$

$\hat{A}_{ML}$ is efficient

while $\hat{A}_{M}$ is not efficient.

Random number generator

$X \sim \text{Pareto}(x_m, A)$

$f(x)$ $\quad U$ is uniform in $[0,1]$

X with p.d.f $f(x)$

$$F(x) = \int_{-\infty}^{x} f(y)\, dy$$

$$F^{-1}(U) \sim X$$

if Y is uniform in $[0, x_m^{-A}]$

Then

$$X = Y^{-\frac{1}{A}} \text{ is Pareto.}$$

U uniform in $[0,1]$

$$X = \left(U x_m^{-A}\right)^{-1/A}$$

$$\mathbb{P}\left(\log \frac{X}{x_m} \leq y\right) =$$

$$\mathbb{P}\left(X \leq x_m e^{y}\right)$$

$$\mathbb{P}(X \leq x) = \int_{x_m}^{x} \frac{A x_m^{A}}{y^{A+1}}\, dy = -\frac{x_m^{A}}{y^{A}} \Big|_{x_m}^{x}$$

$$1 - \left(\frac{x_m}{x}\right)^A$$

$$\mathbb{P}(X \geq x) = \left(\frac{x_m}{x}\right)^A$$

$$\mathbb{P}\left(\log \frac{X}{x_m} \geq y\right) =$$

$$\mathbb{P}\left(X \geq x_m e^{-\;\;y}\right)$$

$$Y = \log \frac{X}{x_m}$$

$$\mathbb{P}(Y \geq y) = \mathbb{P}(X \geq x_m e^{y}) =$$

$$= \left(\frac{x_m}{x_m e^{y}}\right)^A = e^{-Ay}$$

$Y = \log \frac{X}{x_m}$ is exponential with

par A.

Correct answer 1

0

Collect all answers

n_1 of 1

$$p = \frac{n_1}{N}$$

$$p(a) = D + u(a)$$

$$u(a) \sim N(0, \sigma^2)$$

$D \quad \sigma^2$

$D + u(a)$ is a Treshold.

ε_i

$$D + u(a) + \varepsilon \begin{cases} > 0 & 1 \\ < 0 & 0 \end{cases}$$

$$\varepsilon \sim N(0,1)$$

Probit model.

How do I estimate D and σ